Financing Capacity Investment Under Demand Uncertainty

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OPS-MEET-FINANCE

- **OPS**: Capacity investment decisions for new products/markets are often made under demand uncertainty.
  - Trade-off: Excess capacity vs. unmet demand.
  - Problem: Optimize Capacity level.

- Firms investing in capacity must mobilize:
  - Human and physical capital
  - Funds to acquire them.

- **FIN**: When liquidity reserves are low or involve opportunity costs, must tap outside capital markets:
  - Variety of sources: bank loans, equity, etc.
  - Problem: Optimize sourcing of funds.

We study the interplay between the operational and financial facets of capacity investment.
Problem and Approach

- Capacity choice problem under demand uncertainty
  - Newsvendor model

- Limited liquidity

- Access to external funds hampered by financial frictions
  - Moral hazard

How to optimize both capacity and financing choices (debt vs. equity vs. everything else)?

- Optimal contracting approach
  - Available sources of finance derived endogenously
Nascent OPS-meet-FIN literature:
- How does a firm’s funding needs affect its capacity and technology choices?
- How do these choices in turn impact the firm’s financial policy?

Different aspects of operations...
- Capacity choice: static (Dada-Hu 2008, Alan-Gaur MS 2013)
- Capacity choice: dynamics (Li-Shubik-Sobel MS 2013)
- Capacity choice: Flexible vs dedicated techno (Boyabatli-Toktay MS 2011, Chod-Zhou MS 2013)

... but similar models of external financing:
1. An interest rate is set up-front
2. The newsvendor then chooses the loan size

Exception: Chod-Zhou (MS 2013)
This involves two main exogenous restrictions:

1. Focus a priori on debt
2. Assume a priori that interest rates is not conditional on the loan size

Problem:

- Within these models, non-debt financing (e.g. equity) might improve efficiency (even first-best)
- Even if focus on debt, loan size-dependent interest rate might too (Dada-Hu 2008, Prop 4)

=> Risk that recommended distortion of OPS could be avoided by suitable contract

=> Risk that implicit assumptions justifying contract restrictions would actually affect OPS

Optimal contracting approach => Explore the conditions under which deviations from efficient outcomes arise, once feasible contractual solutions are exhausted
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Our work is further connected to:

- Optimal contracts w/ limited liability (Poblete-Spulber Rand 2012, Gromb-Martimort JET 2007)

- Optimal contracts for newsvendor (Dai-Jerath MS 2013)

- Optimal financial contracts under limited liability (Innes JET 1990, Tirole 2006)
Roadmap

▶ Introduction

▶ Model
  ▶ OPS: Newsvendor Model
  ▶ FIN: Financing under moral hazard

▶ Optimal Financing

▶ Optimal Capacity

▶ Conclusion
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Newsvendor Model

- Firm is risk neutral
- Demand $D$ with p.d.f $f_1(\cdot)$ and c.d.f $F_1(\cdot)$
- Set capacity $q$ at unit cost $c > 0$
- If $q > 0$, fixed (non-monetary) opportunity cost $\kappa_1 \geq 0$.
- Revenue $r$ per unit sold, Salvage value $s$ per unsold unit
- Total Revenue, randomly distributed over $[sq, rq]$, p.d.f $g_{1,q}(\cdot)$

\[ P_{1,q} \equiv sq + (r - s) (D_1 \land q) \]

- Expected Profit

\[ \pi_1(q) \equiv \mathbb{E}[P_{1,q}] - cq \]

- Optimal capacity

\[ q_{FB}^1 \equiv \arg\max_{q \in \mathbb{R}_+} \pi_1(q) = F_1^{-1} \left( \frac{r - c}{r - s} \right) \quad \text{if} \quad \pi_1 \geq \kappa_1 \quad \text{else} \quad q_{FB}^1 = 0 \]
Financing

- Firm has internal funds $W$

- Financial contract with risk-neutral, competitive investor:
  - Investment $I$
  - Capacity $q$
  - State-contingent repayment $R(\cdot)$

- Financial contracting:
  1. Demand is not contractible but Total Revenue is -
     $\Rightarrow R(p) : [sq, rq] \mapsto \mathbb{R}_+$
  2. Limited liability (e.g. aversion to negative wealth)
     $\Rightarrow R(p) \leq p$
  3. Monotonicity (e.g. firm can inflate revenue)
     $\Rightarrow$ non-decreasing $R(p)$

- Feasible contracts: Debt, equity, etc.
Financial Frictions - Moral Hazard

- So far, Modigliani-Miller Theorem holds and the financial contracts is irrelevant

- Moral hazard:
  - The firm can work \((e = 1)\) or shirk \((e = 0)\)
  - Effort \(e\) is non-contractible

- Work \((e = 1)\): as above

- Shirk \((e = 0)\):
  - Smaller cost \(\kappa_0 \geq 0\) with \(\Delta \kappa \equiv \kappa_1 - \kappa_0 \geq 0\)
  - Less favourable demand distribution \(f_0(\cdot)\) in the sense of MLRP:
    \[
    f_1/f_0 \text{ is strictly increasing}
    \]

- Assume \(q_1^{FB} \geq 0\) and \(q_0^{FB} = 0\)
The Firm’s Problem

\[
\max_{q,I,R(\cdot)} \mathbb{E} [P_1,q - R(P_1,q)] + W + I - cq - \kappa_1
\]

s.t.

- Firm participation constraint
  \[
  \mathbb{E} [P_1,q - R(P_1,q)] + W + I - cq - \kappa_1 \geq W
  \]
- Funding constraints:
  \[
  \mathbb{E} [R(P_1,q)] \geq I \quad \text{and} \quad I + W \geq cq
  \]
- Feasibility, i.e., limited liability and monotonicity constraints
  \[
  R(p) \leq p \quad \text{and} \quad R(p') \leq R(p) \quad \text{with} \quad p > p'
  \]
- Incentive compatibility constraint (IC) for \( q > 0 \)
  \[
  \mathbb{E} [P_1,q - R(P_1,q)] - \mathbb{E} [P_0,q - R(P_0,q)] - \Delta \kappa \geq 0
  \]

If the previous problem is not feasible then \( q = 0 \).
The Firm’s Problem

\[
\max_{q,l,R(\cdot)} \mathbb{E} [P_{1,q}] + W - cq - \kappa_1
\]

s.t.

- Firm participation constraint

\[
\mathbb{E} [P_{1,q}] - cq - \kappa_1 \geq 0
\]

- Funding constraints:

\[
\mathbb{E} [R(P_{1,q})] = I \quad \text{and} \quad I + W \geq cq
\]

- Feasibility, i.e., limited liability and monotonicity constraints

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R(p) \leq p \quad \text{and} \quad R(p') \leq R(p) \quad \text{with} \quad p > p'
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\]

If the previous problem is not feasible then \( q = 0 \).
The Firm’s Problem

\[
\max_{q,R(\cdot)} \pi_1(q)
\]  

subject to:

- Firm participation constraint

\[
\pi_1(q) - \kappa_1 \geq 0
\]

- Funding constraint:

\[
\mathbb{E}[R(P_1,q)] \geq (cq - W)^+
\]

- Feasibility, i.e., limited liability and monotonicity constraints

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R(p) \leq p \quad \text{and} \quad R(p') \leq R(p) \quad \text{with} \quad p > p'
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- Incentive compatibility constraint (IC) for \(q > 0\)

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If the previous problem is not feasible then \(q = 0\).
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What is the optimal way to finance a given capacity $q$? Feasible contracts are many:

- Debt $R(p) = p \wedge K$
- Equity $R(p) = \alpha p$
- Convertibles, combinations, etc.
Optimality of Debt

Proposition

For a given capacity $q$, if the set of feasible contracts is not empty then the firm exerts effort $e = 1$ and an optimal policy for financing the capacity cost $cq$ is to use its internal funds $W$ and fund any short-fall $l(q) = (cq - W)^+$ with debt with face value $K(q)$ equal to the unique solution to

$$K - (r - s) \int_0^\left(\frac{K - sq}{r - s}\right)^+ F_1(x) \, dx = (cq - W)^+ \quad (4)$$

- If $W \geq cq$ then $K = 0$ and the firm does not raise funds.
- If $cq > W \geq (c - s)q$ then $K = l(q) \in (0, sq]$ and debt is risk free.
- If $(c - s)q > W$, then $K \in (sq, rq]$ and debt involves default risk.
Default probability $\delta(q)$

**Proposition**

Default probability $\delta(q) \equiv Pr (P_{1,q} < K(q))$ is non-decreasing convex (resp. concave) in $q$ if and only if demand $D_1$ has increasing (resp. decreasing) hazard rate.
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Optimal Capacity $q$

$$\max_{q,R(\cdot)} \pi_1(q)$$

s.t.

- Firm participation constraint
  $$\pi_1(q) - \kappa_1 \geq 0$$

- Funding constraint:
  $$\mathbb{E}[R(P_1,q)] \geq (cq - W)^+$$

- Feasibility, i.e., limited liability and monotonicity constraints
  $$R(p) \leq p \quad \text{and} \quad R(p') \leq R(p) \quad \text{with} \quad p > p'$$

- Incentive compatibility constraint (IC) for $q > 0$
  $$\mathbb{E}[P_1,q - R(P_1,q)] - \mathbb{E}[P_0,q - R(P_0,q)] - \Delta \kappa \geq 0$$

If the previous problem is not feasible then $q = 0$. 
Main Result - Theorem 1

The firm’s optimal choice of capacity \( q^* \) are as follows.

- If \( W \geq cq_1^{FB} \) then the firm undertakes the project without raising additional funds, and sets up the first-best capacity \( q^* = q_1^{FB} \).

- If \( cq_1^{FB} > W \geq (c - s) q_1^{FB} \) then the firm issues debt, undertakes the project, and sets up the first-best capacity \( q^* = q_1^{FB} \).

- If \( (c - s) q_1^{FB} > W \) then two thresholds \( \Delta_{\kappa} \) and \( \Delta_{\bar{\kappa}} \) exist such that the optimal outcome is as follows.
  
  i. If \( 0 \leq \Delta_{\kappa} \leq \Delta_{\bar{\kappa}} \) then the firm issues debt, undertakes the project, and sets up the first-best capacity \( q^* = q_1^{FB} \).
  
  ii. If \( \Delta_{\kappa} < \Delta_{\kappa} \leq \Delta_{\bar{\kappa}} \) then the firm issues debt, undertakes the project, and sets up a capacity level that strictly exceeds the first-best level \( q^* > q_1^{FB} \). Further, \( q^* \) is monotonically increasing with \( \Delta_{\kappa} \).
  
  iii. If \( \Delta_{\kappa} > \Delta_{\bar{\kappa}} \) then the firm abandons the project \( (q^* = 0) \).
Optimal Capacity $q^*$ as a function of moral hazard problem’s severity $\Delta \kappa$
Optimality of Overinvesting

Insights

- Not surprising:
  - First best if $W$ is large enough
  - First best if $\Delta \kappa$ is small enough
  - Abandon if $\Delta \kappa$ is large enough

- More surprising: Overinvestment $q^* > q_1^{FB}$
The Impact of Internal Funds on Investment

Proposition

Two non-negative thresholds \( W \) and \( \bar{W} \) exist such that

i. If \( W < \underline{W} \) then the project is abandoned (\( q^* = 0 \)).

ii. If \( W \in [\underline{W}, \bar{W}) \) then capacity exceeds the first best (\( q^* > q_{1FB}^{FB} \)) and decreases with internal funds \( W \).

iii. If \( W \geq \bar{W} \) then the first-best capacity is set up \( q^* = q_{1FB}^{FB} \).
Optimal Capacity $q^*$ as a function of internal funds $W$
Conclusion

- Study the interaction between capacity choice and financing choice problems
  - Relevant only under financial frictions: here moral hazard problem
  - The firm must consider available sources of funds when calibrating its capacity investment

- Main results:
  - Debt financing is optimal for any given capacity level (Innes 1990)
  - Given optimal debt financing
    - The optimal capacity is never below the efficient level
    - Overinvesting can be optimal
    - ‘The higher the internal funds, the lower the investment.’

- Perspective:
  - Optimal contracting approach: Available sources of finance derived endogenously
  - Contrast to most of the OPS-meet-FIN literature => New direction